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Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	9	/	0	1 R	Signature	

Paper Reference(s)

6669/01R

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Monday 24 June 2013 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. The hyperbola H has foci at (5, 0) and (–5, 0) and directrices with equations

$x = \frac{9}{5}$ and $x = -\frac{9}{5}$.

Find a cartesian equation for H.

(7)

Leave blank

2. Two skew lines l_1 and l_2 have equations

$$l_1: \mathbf{r} = (\mathbf{i} - \mathbf{j} + \mathbf{k}) + \lambda(4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$$
$$l_2: \mathbf{r} = (3\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}) + \mu(-4\mathbf{i} + 6\mathbf{j} + \mathbf{k})$$

respectively, where λ and μ are real parameters.

(a) Find a vector in the direction of the common perpendicular to l_1 and l_2

(2)

(b) Find the shortest distance between these two lines.

(5)

P 4 2 9 5 6 A 0 4 3 2

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Question 2 continued

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3. The point P lies on the ellipse E with equation

$$\frac{x^2}{36} + \frac{y^2}{9} = 1$$

N is the foot of the perpendicular from point P to the line $x = 8$

M is the midpoint of PN.

(a) Sketch the graph of the ellipse E, showing also the line $x = 8$ and a possible position for the line PN.

(1)

(b) Find an equation of the locus of M as P moves around the ellipse.

(4)

(c) Show that this locus is a circle and state its centre and radius.

(3)

Leave blank

Question 3 continued

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4. The plane Π_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + s\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix},$$

where s and t are real parameters.

The plane Π_1 is transformed to the plane Π_2 by the transformation represented by the matrix $\mathbf{T}$, where

$$\mathbf{T} = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$

Find an equation of the plane Π_2 in the form $\mathbf{r}.\mathbf{n} = p$

(9)

P 4 2 9 5 6 A 0 1 2 3 2

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Question 4 continued

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5.

$$I_n = \int_1^5 x^n (2x-1)^{-\frac{1}{2}}\,\mathrm{d}x, \qquad n \geqslant 0$$

(a) Prove that, for $n \geqslant 1$,

$$(2n+1)I_n = nI_{n-1} + 3 \times 5^n - 1$$

(5)

(b) Using the reduction formula given in part (a), find the exact value of I_2

(5)

P 4 2 9 5 6 A 0 1 6 3 2

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Question 5 continued

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6. It is given that $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ is an eigenvector of the matrix **A**, where

$$\mathbf{A} = \begin{pmatrix} 4 & 2 & 3 \\ 2 & b & 0 \\ a & 1 & 8 \end{pmatrix}$$

and a and b are constants.

(a) Find the eigenvalue of **A** corresponding to the eigenvector $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$.

(3)

(b) Find the values of a and b.

(3)

(c) Find the other eigenvalues of **A**.

(5)

P 4 2 9 5 6 A 0 2 0 3 2

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Question 6 continued

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7.

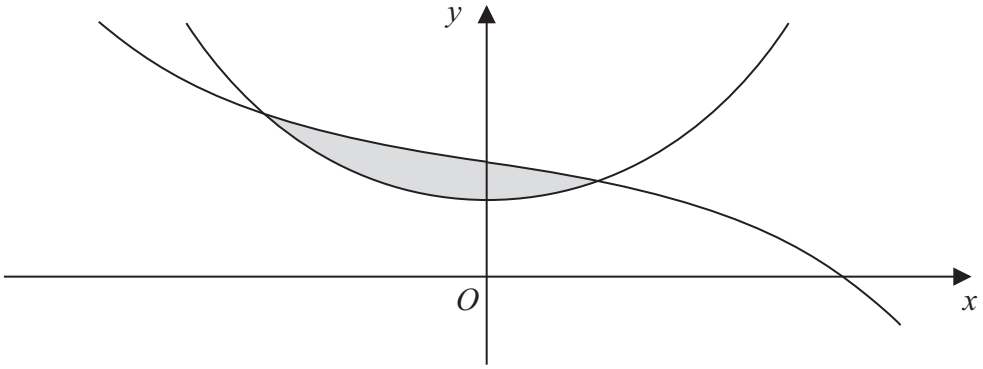

Figure 1

The curves shown in Figure 1 have equations

$$y = 6\cosh x \quad \text{and} \quad y = 9 - 2\sinh x$$

(a) Using the definitions of $\sinh x$ and $\cosh x$ in terms of e^x, find exact values for the x-coordinates of the two points where the curves intersect.

(6)

The finite region between the two curves is shown shaded in Figure 1.

(b) Using calculus, find the area of the shaded region, giving your answer in the form $a \ln b + c$, where a, b and c are integers.

(6)

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Question 7 continued

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8.

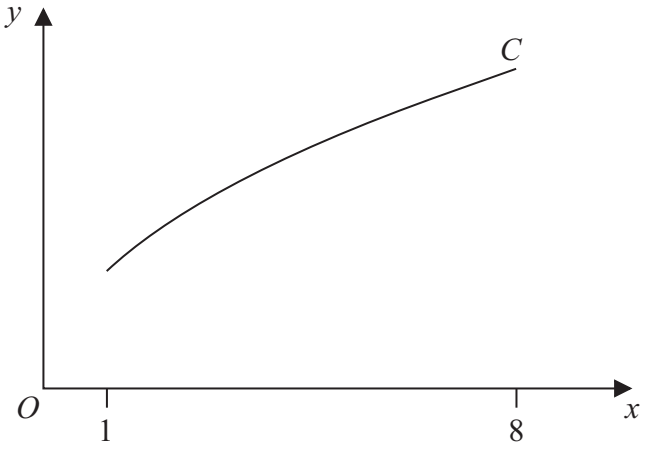

Figure 2

The curve C, shown in Figure 2, has equation

$$y = 2x^{\frac{1}{2}}, \qquad 1 \leqslant x \leqslant 8$$

(a) Show that the length s of curve C is given by the equation

$$s = \int_1^8 \sqrt{\left(1 + \frac{1}{x}\right)}\,\mathrm{d}x$$

(2)

(b) Using the substitution $x = \sinh^2 u$, or otherwise, find an exact value for s.

Give your answer in the form $a\sqrt{2} + \ln(b + c\sqrt{2})$ where a, b and c are integers.

(9)

P 4 2 9 5 6 A 0 2 8 3 2

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Question 8 continued

Q8

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

P 4 2 9 5 6 A 0 3 2 3 2